STBC-Based Transmit Diversity System in Mobile Fading Channel

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ABSTRACT

Space-time block codes (STBC) can obtain full transmit diversity gain in slow-fading environment. But it is not the case for STBC in fast-fading channel. As a result of rapid time variation of the channel, the orthogonality of STBC is lost. This will cause decoding in error and hence the system performance is degraded. In this paper, we consider both coherent and differential detection for STBC and evaluate the performance in mobile fading channel by theoretical analysis and computer simulation, as well. Furthermore, the limitations for STBC-based transmit diversity system are pointed out by analyzing the simulation results.

I. INTRODUCTION

The concepts of space-time codes come from combining transmit diversity and coding technique. There are three major types of space-time codes (STC): space-time block codes (STBC), space-time trellis codes (STTC) and unitary space-time codes (USTC). The requirement of channel estimation for coherent STBC [1][2][8] and STTC [3] is crucial; by contrast, differential STBC [4][5] and USTC [6] can be operated in the mobile fading environment, where the channel state information (CSI) is difficult to obtain. Due to its simplicity and well performance as compared with STTC and USTC, STBC gains more popularity.

STBC has been widely discussed in slow-fading channel, based on the assumption that the channel is constant over a period of time (i.e. one STBC block duration for coherent detection and two STBC block durations for differential detection) and varies from one period to another. In fast-fading channel, where the channel is no longer constant over a period of time, the estimator for coherent STBC cannot trace the channel variation instantaneously. This will cause decoding in error and hence the system performance is degraded. According to the time correlation of the mobile fading process [7], some improvement by applying linear interpolation is studied in [9]. Similar to coherent STBC, differential STBC also suffers from rapid channel variation and loses the orthogonality. It is the loss in orthogonality that causes the degradation in system performance.

In this paper, we consider both coherent and differential detection for STBC and evaluate the performance in mobile fading channel by theoretical analysis and computer simulation, as well. Furthermore, the limitations for STBC-based transmit diversity system are pointed out by analyzing the simulation results. Without loss of generality, we consider the STBC-based transmit diversity system for $g_2$ with two transmit antennas at base and one receive antenna at remote in the wireless downlink transmission.

The rest of this paper is organized as follows. In Section II, the coherent STBC-based transmit diversity system for $g_2$ is reviewed. It is described that the estimator cannot trace the channel variation instantaneously in fast-fading channel, which results in the vanishment of orthogonality. In Section III, the differential STBC-based transmit diversity system for $g_2$ is presented. It is depicted that the loss of orthogonality in fast-fading channel. Finally, the simulation results are shown in Section IV and the conclusion is made in the last section.

II. COHERENT DETECTION

It is known that STBC for multiple transmit antennas obtains the same diversity gain as maximum ratio combining (MRC) for multiple antennas at receiver [2]. But it is not the case when coherent STBC is operated in fast-fading channel, where the channel estimator provides no instantaneous channel estimate for decoding. Hence, the orthogonality is lost and the system performance is degraded. Subsequent analysis for coherent STBC in fast-fading channel is based on two assumption: first, perfect phase recovery...
is the path gain from transmit antenna \( n \) at time \( t \), and is generated by mobile fading channel simulator \([7]\); \( c_{nt} \) is the coded symbol to be transmitted from antenna \( n \) at time \( t \); \( \eta_t \) is the AWGN at time \( t \). Consequently, the system function for \( g_2 \) is \([8]\)

\[
\begin{bmatrix}
r_t \\
r_{t+1}
\end{bmatrix} = \begin{bmatrix}
\alpha_1 & -\alpha_2 \\
\alpha_2 & \alpha_1
\end{bmatrix} \begin{bmatrix}
s_t \\
s_{t+1}
\end{bmatrix} + \begin{bmatrix}
\eta_t \\
\eta_{t+1}
\end{bmatrix},
\]

where \( s_t \) and \( s_{t+1} \) are the uncoded symbols at time \( t \) and \( t+1 \), respectively.

In addition, for \( g_2 \), assume the perfect pilot estimates for both transmit antennas at time \( t \) of each STBC block are available at receiver, i.e.,

\[
\hat{\alpha}_n = \alpha_n, \quad \text{for} \quad n = 1, 2.
\]

Thus, the receiver uses \( \hat{\alpha}_n \) to decode the received signals for both two time slots in each STBC block. Now define \( \epsilon_n \) as the estimation error for the path gain from transmit antenna \( n \) at time \( t+1 \), i.e.,

\[
\epsilon_n = \hat{\alpha}_n - \alpha_{n,t+1}, \quad \text{for} \quad n = 1, 2.
\]

Then, the system function for \( g_2 \) can be rewritten as

\[
\begin{bmatrix}
r_t \\
r_{t+1}
\end{bmatrix} = \begin{bmatrix}
\alpha_1 & -\alpha_2 \\
\alpha_2 & \alpha_1
\end{bmatrix} \begin{bmatrix}
s_t \\
s_{t+1}
\end{bmatrix} + \begin{bmatrix}
\eta_t \\
\eta_{t+1}
\end{bmatrix} + \begin{bmatrix}
0 & 0 \\
-\epsilon_2 & \epsilon_1
\end{bmatrix} \begin{bmatrix}
s_t \\
s_{t+1}
\end{bmatrix},
\]

or

\[
\bar{r} = \bar{H}s + \bar{c} + \bar{\eta}.
\]

Apply matched filtering, the decision vector is obtained as

\[
D = \bar{H}^o \bar{r} = [d, \ d_{t+1}]^T.
\]

Therefore, we have the decision \( d_t \) after decoding \( s_t \),

\[
d_t = \begin{pmatrix}
\hat{\alpha}_1^2 + \hat{\alpha}_2 \alpha_{2,t+1} \\
\hat{\alpha}_2^2 - \hat{\alpha}_1 \alpha_{1,t+1} + \hat{\alpha}_1 \hat{\alpha}_2 \eta_t + \hat{\alpha}_2 \eta_{t+1}
\end{pmatrix} + \text{signal + ISI + noise}
\]

which includes the desired signal, the spatial inter-symbol interference (ISI) from undesired symbol, and the noise term. Similarly, the decision \( d_{t+1} \) after decoding \( s_{t+1} \) can be obtained. From (8), it is clear that ISI results in irreducible error floor.

### III. Differential Detection

Similar to coherent STBC, differential STBC for multiple transmit antennas cannot acquire full diversity gain in fast-fading channel where the channel is no longer constant over two STBC block durations. In other words, this rapid channel variation destroys the orthogonality of STBC. Therefore, the system performance is degraded.

Subsequent analysis for differential STBC in fast-fading channel is made by assuming no CSI are available at receiver; in other words, neither perfect phase recovery nor perfect pilot recovery is applied. The differential encoding algorithm for STBC \( g \), can be seen in \([4]\). Define \( \mathcal{M} \) as a one-to-one mapping which maps 2b bits \( \beta \) onto \( \nu \), which is a set consisting of \( 2^{2b} \) unit-length vectors. The operation of differential encoding can be expressed as

\[
s_{2t+1} = \mathcal{M}(\beta_{2t+1}) \mathcal{S}_{2t+1}^n
\]

or

\[
\mathcal{M}(\beta_{2t+1}) = s_{2t+1} = \mathcal{S}_{2t+1}^n
\]

where

\[
\mathcal{S}_{2t+1}^n = \begin{bmatrix}
s_{2t+1} \\
s_{2t+2}
\end{bmatrix}
\]

and

\[
\mathcal{S}_{2t+1} = \begin{bmatrix}
s_{2t+1} \\
s_{2t+2}
\end{bmatrix}
\]

Initially, at time \( 2t+1 \), \( s_{2t+1} \) and \( s_{2t+2} \) are sent from transmit antenna 1 and 2, respectively; at time \( 2t+2 \), \( s_{2t+2} \) and \( s_{2t+3} \) are sent from transmit antenna 1 and 2, respectively. At time \( 2t+1 \), a block of 2b bits \( \beta_{2t+1} \) arrives at the encoder. The transmitter computes (9) to obtain \( s_{2t+1} \). Then, at time \( 2t+1 \), it transmits \( s_{2t+1} \) and \( s_{2t+2} \) from transmit antenna 1 and 2, respectively; at time \( 2t+2 \), it transmits \( s_{2t+2} \) and \( s_{2t+3} \) from transmit antenna 1 and 2, respectively. The receiver can recover the transmitted information bits without knowing the channel condition beforehand.

In order to decode one block of transmitted information bits, the received symbols of two consecutive STBC blocks are considered simultaneously. These two blocks can be expressed as

\[
r_{2t+1} = [t_{2t+1} \ t_{2t+2}],
\]

where
\( R_{2 \times 1} = \begin{bmatrix} r_{2 \times 1} \\ r_{2 \times 1}^* \end{bmatrix}, \)

where \( r_{2 \times 1} \) and \( r_{2 \times 1}^* \) are the first and the second received block of each decoding block pair, respectively. They can be expressed as

\[
\begin{align*}
\tilde{r}_{2t_1} &= s_{2t_1} + \eta_{2t_1}, \\
\tilde{r}_{2t_1} &= s_{2t_1}^* + \eta_{2t_1}.
\end{align*}
\]

Then, we rewrite (21) as

\[
\begin{align*}
\tilde{r}_{2t_1} &= \sum_{n=1}^{N} \alpha_{2t_1} \tilde{s}_{2t_1} + \eta_{2t_1} \sum_{n=1}^{N} \eta_n, \\
\tilde{r}_{2t_1} &= \sum_{n=1}^{N} \alpha_{2t_1}^* \tilde{s}_{2t_1} + \eta_{2t_1}^* \sum_{n=1}^{N} \eta_n.
\end{align*}
\]

For slow-fading case, the channel is constant over two consecutive blocks as

\[
\Delta_\alpha = \alpha_{2t_1} - \alpha_{2t_1}^*.
\]

Then,

\[
\begin{align*}
\hat{\eta}_1 &= \tilde{r}_{2t_1} - \alpha_{2t_1} \tilde{s}_{2t_1}, \\
\hat{\eta}_1 &= \tilde{r}_{2t_1} - \alpha_{2t_1}^* \tilde{s}_{2t_1}.
\end{align*}
\]

The decoder computes

\[
\begin{align*}
\gamma &= R_{2 \times 1} \alpha_{2 \times 1} - \eta, \\
\gamma &= R_{2 \times 1}^* \alpha_{2 \times 1}^* - \eta.
\end{align*}
\]

Accordingly, by substituting (30) into (21), we obtain

\[
\begin{align*}
\Omega &= \sum_{n=1}^{N} \alpha_{2t_1} \tilde{s}_{2t_1} + \eta_n, \\
\Omega &= \sum_{n=1}^{N} \alpha_{2t_1}^* \tilde{s}_{2t_1} + \eta_n.
\end{align*}
\]

IV. SIMULATION RESULTS

Here, we consider the STBC-based transmit diversity system for \( g \), with two transmit antennas at base and one receive antenna at remote in the wireless downlink transmission. For comparison, the simulation results for both coherent and differential detection are combined together with various normalized Doppler frequency \( f_d T \), where \( f_d \) is the maximum Doppler frequency and \( T \) is the symbol duration. Fig. 1 and Fig. 2 are the system performance for BPSK and 4PSK, respectively, in Rayleigh fading channel. The solid-lines and dot-lines are the curves for coherent and differential detection, respectively. Also, the solid-lines without points are the theoretical curves in slow-fading channel, and the dot-lines without points are the simulation results in slow-fading channel. From both figures, for slow-fading cases, it can be easily seen that the results for differential detection (dot-lines) is about 3dB worse than the ones for coherent detection (solid-lines). With little complexity and well performance, this truly differential detection scheme for STBC is a remarkable contribution from Tarokh.
According to the distinct detection schemes, the reasons that cause performance degradation can be classified into two categories: first, for coherent detection, the orthogonality is vanished, as a result that the channel estimator cannot provide instantaneous channel estimate for decoding; second, for differential detection, the rapid channel variation destroys the orthogonality.

In both figures, the simulation results for fast-fading channel are shown with different channel condition as different normalized Doppler frequency $f_dT = 0.05, 0.02, 0.01, 0.005$, respectively. The higher $f_dT$, the faster time variation of channel. For coherent detection, the curves for $f_dT = 0.01$ is well approached to the ones for slow-fading case. By contrast, for differential detection, instead of the curves for $f_dT = 0.01$ but for $f_dT = 0.005$ is approached to the ones for slow-fading case. This is due to that the coherent detection scheme only requires the channel constant over one STBC block duration while the differential detection technique needs the channel constant over two STBC block durations; in other words, the channel variation has more impact on differential STBC-based transmit diversity system.

For a specific QoS class with BER requirement of $10^{-3}$ at SNR 25dB, the coherent system using BPSK (Fig. 1) or 4PSK (Fig. 2) can be accepted if the normalized Doppler frequency $f_dT$ is not greater than 0.05. On the other hand, for the same QoS class, the differential system using BPSK or 4PSK is acceptable only if the normalized Doppler frequency $f_dT$ is not greater than 0.01. Again, from the difference between the limitations for both coherent and differential STBC, it is definite that the differential STBC has poor performance. But, one should know that the complexity of differential STBC is much lower than that of coherent STBC.

Also, under the same spectral efficiency (i.e. the curves for BPSK with $f_dT = 0.01$ and 4PSK with $f_dT = 0.02$), one can find that the system using BPSK outperforms than the one using 4PSK. This is reasonable, since 4PSK system is of smaller bandwidth compared with BPSK system; in other words, the channel variation for 4PSK system is more serious than for BPSK system.

V. CONCLUSION

It is known that STBC for multiple transmit antennas suffers from performance degradation when operated in fast-fading channel. In our work, the reasons that cause this performance degradation for both coherent and differential detection scheme are depicted theoretically. Also, by analyzing the simulation results, the limitations for STBC-based transmit diversity system in mobile fading channel are pointed out. These allow the system designers to make an easy decision on employing coherent or differential STBC. Furthermore, these can be a reference for combining STBC with other advance transmission technique, such as OFDM and CDMA.

REFERENCES

Fig. 1 Performance of STBC $g_2$ using BPSK modulation in Rayleigh fading channel

Fig. 2 Performance of STBC $g_2$ using 4PSK modulation in Rayleigh fading channel
摘要

在慢速衰落环境中，塊狀時空碼可以獲得完全之傳送分集增益。然而，當其操作在快速衰落通道中，情況卻非如此。由於通道的快速變化，塊狀時空碼之正交性喪失了。這將導致解碼錯誤，進而使得系統效能下降。本論文同時考慮同調塊狀時空碼及非同調塊狀時空碼，並且藉由理論分析及電腦模擬評估其在快速衰落通道中之系統效能。再者，借由分析模擬結果，本論文清楚說明基於塊狀時空碼之傳送分集系統之限制。